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INTERACTING STREAMING NUCLEAR MATTER SYSTEMS IN A RELATIVISTIC QUANTUM FIELD THEORY

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Interacting Streaming Nuclear Matter Systems in a Relativistic Quantum Field Theory*

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Abstract:

We study the energetical behavior of two mutually interacting streaming nuclear matter systems in a relativistic quantum field theory proposed by J.D. Walecka. We show that compared to mutually noninteracting systems the flow velocity is significantly reduced due to the repulsive ω-meson field. We discuss the effective potential between the matter systems.

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As soon as nuclei overcome the Coulomb barrier in a heavy ion collision, they begin to interact by the strong nucleon-nucleon force. For energies up to $E_{c.m.}/A = 20$ MeV, the time-dependent Hartree-Fock model[1] gives a detailed description of the collision dynamics. At the beginning of the reaction process the single particle wavefunctions stream through each other and thus this stage of the reaction essentially corresponds to two streaming mutually interacting nuclear matter systems.

We want to address ourselves to such a situation and study the change in flow velocity caused by the mutual interaction. In the early stage of the reaction the mean potential determines the behavior of the collision dynamics and its determination is essential for gaining some insight into the ongoing process.

Conventional many-body calculations[2], which employ the static potential description of the nucleon-nucleon interaction, are of uncertain reliability when applied to flow velocities close to the velocity of light. Therefore, we treat the problem in a relativistic quantum field theory based on interacting baryon and meson fields.

We consider two equal homogeneous systems of spin and isospin symmetric nuclear matter. Both systems are of infinite size, overlap completely in coordinate space, and stream in z-direction with velocities v and -v, respectively. The interaction between the baryons is assumed to be mediated by two chargeless mesons, the sigma ($\underset{\sim}{\sigma}$) and the omega ($\underset{\sim}{\omega}_\mu$) meson. The corresponding Lagrangian density is

$$L = -\bar{\psi}(\gamma_\mu \partial_\mu + m)\psi - \frac{1}{2}(\partial_\mu \underset{\sim}{\sigma})^2 - \frac{1}{2}\, m_s^2 \underset{\sim}{\sigma}^2 - \frac{1}{4}(\partial_\mu \underset{\sim}{\omega}_\nu - \partial_\nu \underset{\sim}{\omega}_\mu)^2$$

$$- \frac{1}{2}\, m_v^2 \underset{\sim}{\omega}_\mu \underset{\sim}{\omega}_\mu - g_s \bar{\psi}\psi \underset{\sim}{\sigma} + i g_v \bar{\psi}\gamma_\mu \psi \underset{\sim}{\omega}_\mu \tag{1}$$

γ_μ are the usual γ matrices, g_s and g_v are Yukawa coupling constants, m_s and m_v the scalar and vector masses, and m the nucleon mass. ψ is the baryon field. J.D. Walecka[3)] proposed to linearize the corresponding field equations by replacing the meson field operators $\underset{\sim}{\sigma}$ and $\underset{\sim}{\omega}_\mu$ by their expectation values σ and ω_μ. Recent successes in describing the properties of finite nuclei demonstrate the validity of this approach.[4-6)] Because of the symmetry in our problem, there is no net baryon current in z-direction, and all space components of the vector meson field ω_μ vanish. Applying the mean field approximation, the field equations are reduced to a simple form

$$\left\{\gamma_\mu \partial_\mu + m^* + \gamma_0 g_v \omega_0\right\} f_j = 0 \tag{2a}$$

$$m_s^2 \sigma = -g_s \langle \phi | \bar{\psi}\psi | \phi \rangle \tag{2b}$$

$$m_v^2 \omega_0 = g_v \langle \phi | \psi^+ \psi | \phi \rangle \tag{2c}$$

In the Dirac equation, the effective mass $m^* = m + g_s\sigma$ is introduced. The streaming mutually interacting nuclear matter systems are assumed to be described by an antisymmetric, relativistic product many-body state $|\phi\rangle$. The Dirac field operator ψ is expanded in momentum-space by means of plane wave spinors f_j. In the respective rest frame of each system (primed) half of the momentum vectors $\vec{k}'$ are assumed to occupy a Fermi sphere of radius K_F. They appear in the center of mass frame as

$$\vec{k}_\perp = \vec{k}'_\perp \tag{3}$$

$$k_z = \gamma k'_z + \beta\gamma\varepsilon_{k'}$$

where

$$\beta = \pm v \quad ; \quad \gamma = (1 - \beta^2)^{-1/2} \tag{4}$$

In the rest frame of each system there is a nonvanishing net baryon current leading to the single particle energy

$$\varepsilon_{k'} = \gamma g_v \omega_o + [k_\perp'^2 + (k_z' + \beta\gamma g_v \omega_o)^2 + m^{*2}]^{1/2} \tag{5}$$

As we use the mean field approximation and suppose time independent meson fields, there is no dissipative mechanism that might lead to a change of the flow velocity v.

For small values of the velocity v the Fermi spheres overlap leading to doubly occupied single baryon states and a vanishing many body state. To avoid this difficulty we rearrange the doubly occupied cells by blowing up each Fermi sphere[7] as shown in Fig. 1. Since our simple model neglects dynamical aspects, the rearrangement is not unequivocal. With the above rearrangement prescription for $v = 0$, however, the many-body state corresponds to the physically meaningful situation where the baryon states are filled up to a momentum $\tilde{K}_F = 2^{1/3} K_F'$. For small velocities v the integrations necessary to evaluate the source terms in Eqs. (2b) and (2c) are performed approximately by integrating over a Fermi sphere of radius $\tilde{K}_F$. The error introduced by taking a too-large integration volume is corrected by a factor $1 - \alpha$, where α is the ratio of the additional $\vec{k}'$-volume to the total sphere volume. For calculating the source terms one has to take into account the normal ordering of the baryon and antibaryon creation and annihilation operators as discussed in ref. 3).

The field eq. (2b) yields a transcendental equation for the effective mass m*

$$m^* - m + (1 - \alpha)\,\frac{g_s^2}{m_s^2}\,\frac{\gamma}{\pi^3}\int^{\tilde{K}_F} d^3k' \frac{m^*}{\gamma\varepsilon_{k'} + \beta\gamma k_z' - g_v\omega_o} = 0 \tag{6}$$

where for separated Fermi spheres (i.e., $K_F \lesssim \beta\gamma m$) $\alpha = 0$ and $\tilde{K}_F = K_F$.

For the ω_0 meson field we simply get

$$g_v\omega_0 = 2\gamma\rho_0 \frac{g_v^2}{m_v^2}, \text{ where } \rho_0 = \frac{2}{3\pi^2} K_F^3 \tag{7}$$

The energy per nucleon E/N of the streaming interacting matter systems is given by one component of the stress tensor.[3] For E/N in terms of β, $\tilde{K}_F$, ρ_0, and m^* we obtain

$$\frac{E}{N} = \gamma \frac{1-\alpha}{2\pi^3\rho_0} \mathcal{J} + (\gamma^2 - \frac{1}{2}) \frac{g_v^2}{m_v^2} 2\gamma\rho_0 + \frac{1}{2} \frac{m_s^2}{g_s^2} \frac{(m^*-m)^2}{2\gamma\rho_0} - m \tag{8}$$

where

$$\mathcal{J} = \int^{\tilde{K}_F} d^3K' \left[k'^2_\perp + (k'_z + \beta\gamma \frac{g_v^2}{m_v^2} 2\gamma\rho_0)^2 + m^{*2}\right]^{1/2} \tag{9}$$

In order to present the behavior of streaming matter systems transparently and in a way comparable to nonrelativistic many-body theories we want to split the total energy into a kinetic and a potential part. This is accomplished by identifying each system with a free nucleon gas in an effective spatially constant potential. Again, the nucleons in the rest frame of each respective system occupy a Fermi sphere of radius K_F if $\alpha = 0$. Thus the kinetic energy per nucleon is

$$\frac{T}{N} = \gamma \frac{1-\alpha}{2\pi^3\rho_0} \int^{\tilde{K}_F} d^3K' \sqrt{K'^2 + m^2} - m \tag{10}$$

and the effective potential per nucleon is given by

$$\frac{U}{N} = \frac{E}{N} - \frac{T}{N} \tag{11}$$

We assume that before the two matter systems flowed through each other, both systems existed without any spatial overlap in their ground states as given by the Walecka model and approached each other with c.m. kinetic energy per nucleon $T_{c.m.}/N$. We further assume that letting the system completely overlap and interact does not lead to internal excitations in each of the respective systems. Energy conservation then simply gives, considering the two separated systems being of saturation density ρ_0.

$$\frac{T_{c.m.}}{N} - \frac{T}{N} = \frac{U}{N} - \frac{E_0}{N} = \frac{\overline{U}}{N} \tag{12}$$

Here E_0/N is the energy per nucleon of saturated static nuclear matter, $E_0/N = -15.7$ MeV. The potential difference $\overline{U}/N$ determines the change in kinetic energy and flow velocity caused by the mutual interaction of the streaming nuclear matter systems.

For our model calculation we take the parameters from ref. 5). They are $(g_s/m_s)m = 17.95$ and $(g_v/m_v)m = 15.60$. In Fig. 2 we plot the effective potential difference per nucleon $\overline{U}/N$ versus the asymptotic c.m. kinetic energy per nucleon $T_{c.m.}/N$. The arrow indicates where the Fermi spheres of the interacting systems just touch. The curve does not start at zero c.m. kinetic energy since for zero flow velocity, energy is necessary to rearrange the overlapping Fermi spheres. The striking feature of our result is that, for streaming matter systems of normal nuclear matter density, a large amount of initial kinetic energy is converted into potential energy and the flow velocity is, therefore, significantly reduced. In Fig. 3 we show the ratio $R = (T/N)/(T_{c.m.}/N)$. Up to $T_{c.m.}/N = 250$ MeV there is a fast change of this ratio and at $T_{c.m.}/N = 200$ MeV already more than 60% of the initial kinetic energy is converted into repulsive potential energy. We also plot the ratio $\bar{R}$ = v_i/v_n where v_i and v_n are the flow velocities of the interacting and noninteracting systems, respectively. Over a wide range of energy $\bar{R}$ is about 0.6. The conclusion of our investigation is that for high flow velocities the ω-meson field evokes a strong repulsive interaction between streaming systems of normal nuclear matter density. This is mainly due to the four-vector character of the ω-field, which leads to terms proportional to γ^3 in the expression for the energy per nucleon and thus to strong repulsive forces.

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Figure captions

Fig. 1 The rearrangement in $\vec{k}$-space for small flow velocities for the system moving with velocity + v. The shaded volume on the left is transferred to the shaded shell on the right.

Fig. 2 The effective potential energy difference per nucleon $\overline{U}/N$ versus the asymptotic c.m. energy per nucleon $T_{c.m.}/N$. Initial densities for the nuclear matter systems are $\rho_0 = 0.1625\ fm^{-3}$.

Fig. 3 The ratios R and $\overline{R}$, which determine the relative change in the kinetic energies and the relative change of the flow velocities versus the asymptotic c.m. energy per nucleon $T_{c.m.}/N$. The considered matter densities are as in Fig. 2.

References

1) P. Bonche, S. Koonin and J.W. Negele, Phys. Rev. C,13 (1976) 1226

2) K.-H. Müller, Z. Physik A295 (1980) 79

3) J.D. Walecka, Ann. Phys. 83 (1974) 491

4) F.E. Serr and J.D. Walecka, Phys. Lett. 79B (1978) 10

5) J. Boguta, LBL-11894 preprint (December 1980), submitted to Nucl. Phys. A.

6) K.-H. Müller, LBL-12667 preprint (May 1981), submitted to Nucl. Phys.

7) F. Beck, K.-H. Müller and H.S. Köhler, Phys. Rev. Lett. 40 (1978) 837

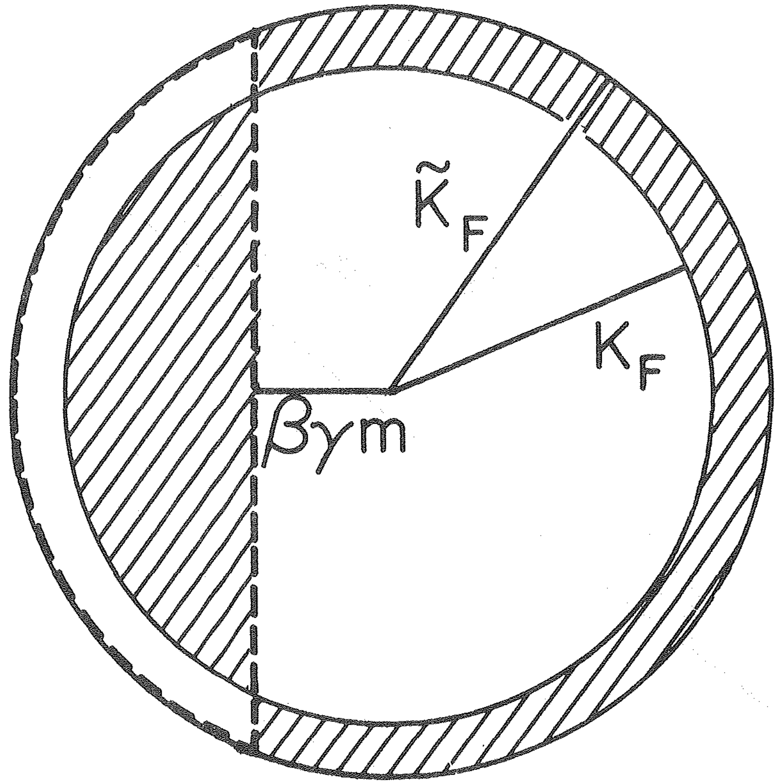

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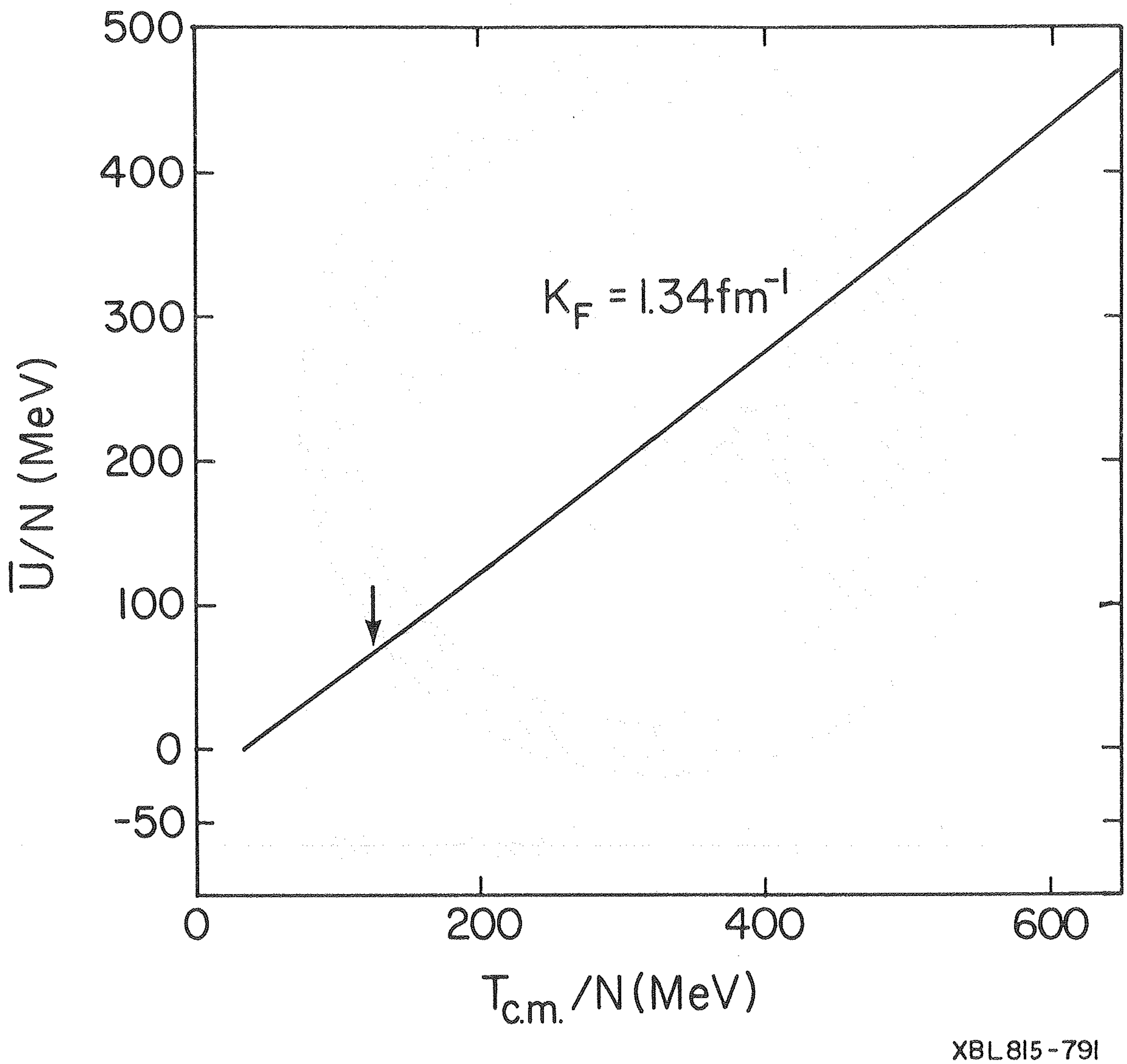
Fig. 1



500
400
300
200
100
0
-50
$\overline{U}/N$ (MeV)
$K_F = 1.34 fm^{-1}$
0
200
400
600
$T_{c.m.}/N$ (MeV)
XBL 815-791

Fig. 2

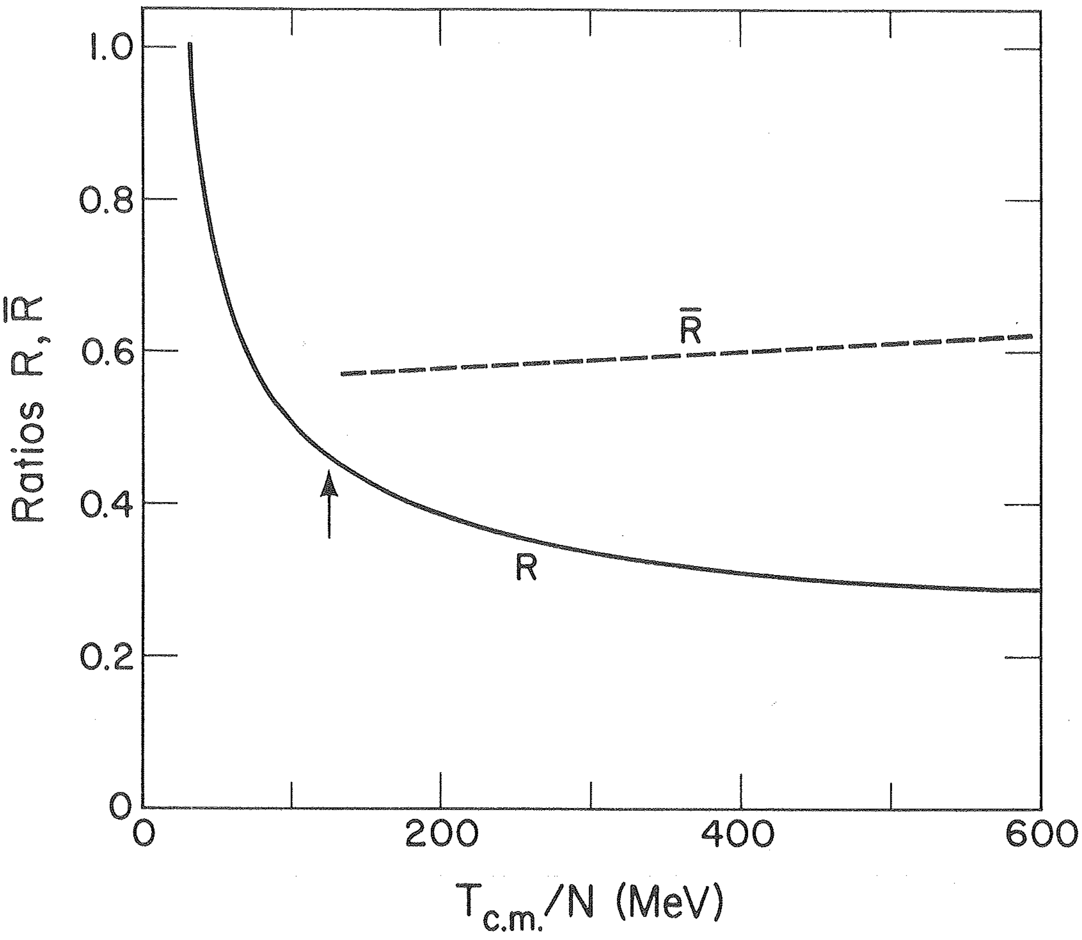


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Fig. 3